Modelling Point-in-time Probability of Default for Macroeconomic Stress Testing

An approach for stress testing credit risk

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8/16/2019

Abstract

Macroeconomic stress testing plays an integral role in the risk management framework of commercial banks. However, it is also deployed as a pivotal tool in the assessment of financial stability by central banks and regulators around the world. As a result, particular emphasis is placed on developing tools for modelling the impact of macro shocks on common risk parameters such as probability of default (PDs) and loss given default (LGDs). In this paper we explore the potential of using a South African economic conditions index (ECI) to transform through-the-cycle (TTC) PDs into point-in-time (PIT) PDs via the Vasicek asymptotic single factor model framework. The ECI is derived using the impulse response functions of a Vector Autoregressive model as in Gaglianone and Areosa (2016). The study shows that the PIT PDs are highly sensitive to the methodology used in the calculation of the systematic risk factor, which has practical implications for how regulators and banks approach their stress testing frameworks.

JEL Classifications: G21, C54, C59

Key words: Credit Risk, Stress Testing, Point-in-time PDs; Through-the-cycle PDs; Vasicek; systematic risk factor

<u>Preliminary and incomplete. Please do not quote.</u>

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1. Introduction

In order to effectively protect the stability of the financial system it is vital to find effective means of assessing the banking sector's vulnerability to macroeconomic shocks. These vulnerabilities can be identified through stress testing the banking system as a whole (macro-prudential stress testing) or individual banks (micro-prudential stress testing). Stress testing is a risk management tool used by banks to assess how portfolio values and capital adequacy ratios change due to extreme but plausible economic conditions. Since the global financial crisis, bank regulators across the globe have also elected to subject banks to macro-prudential stress tests to assess their resilience to shocks.

Banks are typically exposed to a variety of risks, with credit, liquidity and market risks being the most prominent. While the other risks are important, our current focus is on credit risk stress testing. It is vital for both banks and regulators to develop models which can explain the drivers of credit risk such as probability of default (PDs), loss given default (LGDs) and exposures at default (EADs). The PDs reflect the likelihood of a counterparty or borrower defaulting and depends on counterparty specific fundamentals as well as general macroeconomic factors. Two different PD estimates, Through-the-Cycle (TTC) and Point-in-Time (PIT) PDs are used. TTC PDs are those that are largely insensitive to the prevailing macroeconomic conditions while the PIT PDs vary with the economic cycle. They are also commonly referred to as unconditional and conditional PDs respectively. There are particularly significant implications for banks on the Internal Ratings Based Approach (IRB) in the choice of the above estimates in modelling their capital and credit impairments. Inter alia, IRB banks will experience cyclical capital requirements if the chosen PD varies with the economic cycle, thus the proliferation of TTC PDs. The implications of pro-cyclical capital requirements have been discussed at length in Flores *et al*. (2010). Hamilton, (2011) states that's the stability of TTC risk measures comes at the cost of reduced timeliness and default prediction accuracy relative to PIT risk measures.

Since the objective of a credit risk stress tests is to quantify the impact of macro shocks to the portfolio values and capital, the estimation of PIT PDs is vital. Hallblad (2014) discussed the relationship between the TTC and PIT PDs in a Vasicek framework and used the Hodrick-Prescott filter to estimate the economic cycle. Sebolai (2014) used historical default frequencies to estimate the cycle index. In this paper we investigate the transformation of TTC PDs into PIT PDs using the Vasicek asymptotic single factor framework where the systematic risk factor or economic conditions index (ECI) is derived using the impulse response functions of a Vector Autoregressive model as in Gaglianone and Areosa (2016

2. Methodology for calculating Point-in-time PDs

In order to estimate the PIT PDs from the TTC PDs we use the Vasicek single factor methodology. In this framework, an obligor will default on their commitment if the value of the underlying assets falls below a certain threshold (Merton (1974)). Another way of expressing the previous statement is to say that an obligor will default if their credit quality (X_i) falls below a certain threshold, d_i. The credit quality of the obligor is assumed to be influenced by a common systematic factor Y and an idiosyncratic factor Z_i:

$$X_i = Y\sqrt{\rho} + Z_i\sqrt{1-\rho},$$

where ρ is the sensitivity of each obligor to the systematic and idiosyncratic factors and $Y, Z_i \sim N(0,1)$ are mutually independent standard normal variables. Thus the unconditional or cycle-neutral default probability, PD_i is given by:

$$PD_i = P(X_i < d_i) = \Phi(d_i),$$

where Φ is the cumulative distribution function of the standard Gaussian distribution. By inverting the above equation, the threshold can be expressed as a function of the unconditional PD:

$$d_i = \Phi^{-1}(PD_i).$$

Given a particular realization y for the macroeconomic state factor Y, the conditional or PIT PD, PD(y), can be calculated by

$$PD_i(y) = P(X_i < d_i | Y = y)$$

$$= P\left(Y\sqrt{\rho_i} + Z_i\sqrt{1-\rho_i} < d_i \middle| Y = y\right)$$

$$= P\left(Z_i < \frac{d_i - y\sqrt{\rho_i}}{\sqrt{1-\rho_i}}\right)$$

$$= \Phi\left(\frac{d_i - y\sqrt{\rho_i}}{\sqrt{1-\rho_i}}\right)$$

$$= \Phi\left(\frac{\Phi^{-1}(PD_i) - y\sqrt{\rho_i}}{\sqrt{1-\rho_i}}\right)$$

Therefore, the PIT PD is a function of the TTC PD, a systematic risk factor and the correlation parameter. The value of the systematic risk or economic state factor reflects the position of the economy in the economic cycle at a given point in time. Since the economic state factor is assumed to be standard normally distributed, the value of this variable is negative when the economic condition is worse than the equilibrium or "normal" state, and positive when the economic environment is better than the normal state.

Under Basel III the systematic risk factor prescribed for regulatory capital is $\Phi^{-1}(0.999) = 3.09$ which means the regulatory capital should be sufficient to absorb a loss with a probability of 1 in 1000. The correlation parameter is also prescribed for IRB banks under Basel III for each asset class and empirical results show that the correlation varies with the PD. The next section explores the estimation of the Economic Conditions Index (ECI) or systematic risk factor for South Africa

3. Methodology for constructing the ECI for South Africa

The methodology employed to produce the ECI is similar to the one used by Gaglianone and Areosa (2016) in calculating the Financial Conditions Index (FCI) for

Brazil. Here the authors estimated the weights that are used to calculate the FCI from a Vector Autoregression (VAR) model, based on an economic activity proxy. Thereafter impulse-response functions (IRF) were constructed and the weights are given by the twelve-month accumulated response of the economic activity proxy, resulting from shocks in the remaining explanatory variables. The following steps were followed in this paper to calculate the ECI using the VAR approach.

3.1. Data Description

A set of 40 variables is used for the estimation of the South Africa's macroeconomic state. These variables are grouped into eight segments that represent various sectors of the macro-economy that can ostensibly affect bank credit risk (see Table 1). While these variables potentially do not encapsulate all the segments of the economy, it is anticipated that they provide adequate information for the development of an ECI, which in turn will be used as a parameter for transforming TTC PDs to PIT PDs.

Table 1 Selected variables groupings

Variable groupings	Variables
Macroeconomic indicators (Macro)	CPI, Formal non-agricultural employment, Unemployment rate, Household consumption expenditure, Gross fixed capital formation, M3 money supply (M3)
Lending rates indicators (Lend)	Repo rate (Repo) , Non-mortgage lending rate, Mortgage lending rate, Long term government bond yields, Short term effective lending spread, Long term effective lending spread
Deposit rates indicators (Dep)	Short term deposit rate, Long term deposit rate, Short term deposits, Long term deposits
Credit indicators (Credit)	Broad credit, Private sector credit, Mortgage Private sector credit, Non-mortgage Private sector credit
Exchange rate indicators (ER)	Nominal effective exchange rate, Dollar/Rand, Dollar/Euro
External sector indicators (Ext)	Oil prices, Commodity prices, Foreign wholesale prices, Global growth, Current account balance to GDP
Wealth indicators (Wealth)	All share index, House price index, Net household savings to disposable income, Household disposable income, Net household wealth, Household assets
Debt indicators (Debt)	Household debt to disposable income, Household debt to GDP, Household debt, Government debt to GDP, Household financial liabilities

The time series dataset used in calculating the South Africa's ECI is taken from the South African Reserve Bank Quarterly Bulletin. The data frequency is quarterly, spanning the period from the first quarter of 1994 to the fourth quarter of 2018. The sample period was determined by the availability of data used. Except where data is presented as an interest rate or a ratio to GDP, the series have been transformed to year-on-year growth rates. Furthermore, in order to ensure that variables are measured in a comparable scale and use a common distribution, each variable has been standardised to generate *z-scores*. This is done by subtracting the mean and dividing by the standard deviation of each variable.

3.2. Testing for Stationarity

The non-stationary nature of time series data can lead to spurious regressions and misleading results when conventional empirical analysis is used. Therefore, it is important to test time series for a unit root and take appropriate steps should it the series be non-stationary. The stationarity test used in this paper is the Augmented Dickey-Fuller unit root test (ADF) suggested by Said and Dickey (1984). Unit root tests are conducted for all individual variables and on the constructed indicators. When data is non-stationary and needs to be differenced d-times to be made stationary, it is said to be integrated of order d $[I(d)]$. A rule of thumb for this paper is to exclude any variable or indicator that is non-stationary after first differencing.

Since the VAR(p) model requires that the variables used in the models be stationary to avoid spurious results, stationarity testing is a pre-requisite. The results of the unit root tests are shown and discussed in Appendix 1. Table A and B in Appendix 1 shows that all variables and indicators are stationary at either untransformed levels or when differenced once, making them integrated of order 0 and 1. Therefore, based on the fact that the null hypothesis is rejected for all the variables either with or without trend or for both cases, at their levels or after first differencing, none of the variables or the indicators are excluded from the calculation of the indicators.

3.3. Creating indicators using principal components analysis

In order to ensure that as many variables as possible are included in the ECI without affecting the degrees of freedom of the VAR model, variables are grouped to create indicators, which are then used to estimate the VAR model, create the IRFs and ultimately calculate the weighted ECI. Principal component analysis (PCA) is used to systematically reduce the number of variables to a smaller set of components that still explain a large amount of the variability in the original variables[1]. This is done by first grouping the variables into pre-determined groups and then performing PCA on each variable grouping to create a single factor that encapsulates all the included variables.

For this paper, the number of principle components is chosen such that they account for at least 75% of the original total variance. Therefore, each indicator is calculated as follows[2]:

$$Macro_t = \sum_{i=1}^{3}\left[\sum_{k=1}^{6}(PC_{ik} \times X_{k,t})\right]$$

$$Lend_t = \sum_{i=1}^{3}\left[\sum_{k=1}^{6}(PC_{ik} \times X_{k,t})\right]$$

$$Dep_t = \sum_{i=1}^{2}\left[\sum_{k=1}^{4}(PC_{ik} \times X_{k,t})\right]$$

$$Credit_t = \sum_{i=1}^{3}\left[\sum_{k=1}^{4}(PC_{ik} \times X_{k,t})\right]$$

[1] Although the total number of variables, *p*, are needed to produce the total system variability, according to Mingione (2011), much of this variability can be reproduced by a small number, *k*, of the principal components given that "there is (almost) as much information in the *k* components as there is in the original *p* variables".

[2] The indicators are calculated as the products of the values observed for the each variable and each component for each period. The principal component scores are then added together to create the indicator that will be used in the VAR model. For example, the principal component scores for the first principal component for the macro indicator are calculated by multiplying each variable in the macro group by the first principal component of each variable and adding the results together. This is done for each time period. Since the number of principal components for Macro was chosen to be three, this would be done three times to obtain three principal component scores for each time period which are then added to create one indicator. The same method is then followed for all other indicators.

$$ER_t = \sum_{i=1}^{2} \left[\sum_{k=1}^{3} (PC_{ik} \times X_{k,t}) \right]$$

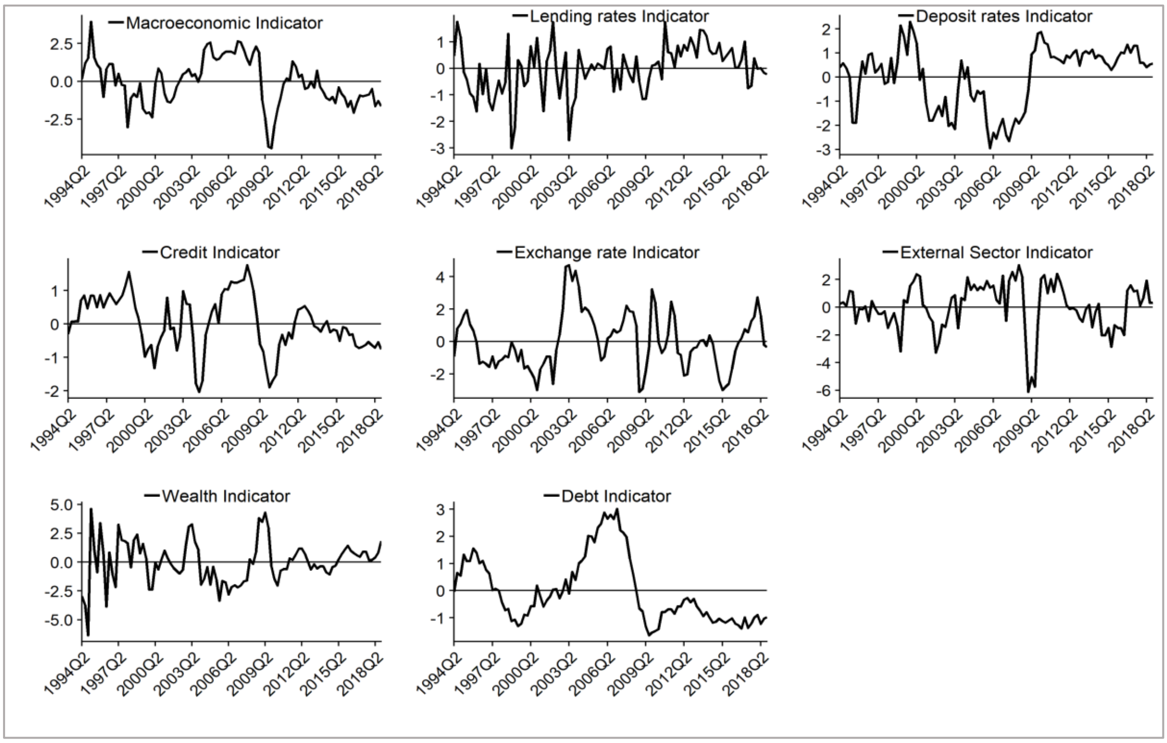
$$Ext_t = \sum_{i=1}^{2} \left[\sum_{k=1}^{5} (PC_{ik} \times X_{k,t}) \right]$$

$$Wealth_t = \sum_{i=1}^{3} \left[\sum_{k=1}^{6} (PC_{ik} \times X_{k,t}) \right]$$

$$Debt_t = \sum_{i=1}^{2} \left[\sum_{k=1}^{5} (PC_{ik} \times X_{k,t}) \right]$$

where k is the number of variables in each indicator, i is the number of principal components, PC is a (i × 1)-matrix of principal components and X = (x1, x2, ..., xk) is a (1 × k)-matrix of variables. Figure 1 depicts the indicators calculated using the above equations.

Figure 1 Economic conditions index indicators



3.4. Vector autoregressive (VAR) model

The factors derived from the PCA analysis are then modelled in a VAR setup. The VAR model used in this paper is of the following form:

$$Y_t = \beta_0 + \sum_{i=1}^{\rho} \theta_i Y_{t-i} + \sum_{j=0}^{\rho} \pi_{k,t-j} Z_{k,t-j} + \epsilon_t, t = 1, 2, 3, ..., T \tag{1}$$

where k is the number of regressors in each model and β0 is a $(k + 1)$-vector of intercepts. The π's are $(k + 1)$ $(k + 1)$-matrices of explanatory variable coefficients.

While the VAR model estimates k equations with each of the k regressors acting as a dependent variable in one equation, this paper will focus on the equation where RGDP is the dependent variable. Therefore, the paper estimates the following VAR(ρ) equation:

$$\begin{aligned} RGDP_t = \beta_0 + \sum_{i=1}^{\rho} \theta_i RGDP_{t-i} + \sum_{j=0}^{\rho} \pi_{1,t-j} Macro_{t-j} + \sum_{j=0}^{\rho} \pi_{2,t-j} Lend_{t-j} + \\ \sum_{j=0}^{\rho} \pi_{3,t-j} Dep_{t-j} + \sum_{j=0}^{\rho} \pi_{4,t-j} Credit_{t-j} + \sum_{j=0}^{\rho} \pi_{5,t-j} ER_{t-j} + \sum_{j=0}^{\rho} \pi_{6,t-j} Ext_{t-j} + \\ \sum_{j=0}^{\rho} \pi_{7,t-j} Wealth_{t-j} + \sum_{j=0}^{\rho} \pi_{8,t-j} Debt_{t-j} + \epsilon_t \end{aligned} \tag{2}$$

where ρ is the model maximum lag order and is in turn determined using the Akaike information criterion (AIC).

3.5. Impulse response functions based of VAR model

One of the benefits of estimating a VAR model is the ability to easily construct IRFs. IRFs are used to track the effects of a one-time exogenous shock or innovation in one of the system's variables on some (or all) of the other variables over a certain period of time. That is, a shock to the *k*-th variable not only directly affects the *k*-th variable but is also transmitted to all of the other endogenous variables through the lag structure of the VAR.

In order to find the impulse response of a VAR(p) model, it must be noted that any stable VAR(p) possesses an MA(∞) representation[3]:

$$Y_t = \bar{Y} + \sum_{j=0}^{\infty} \Psi_j \epsilon_{t-j} \tag{3}$$

where:

$$[\Psi_n]_{l,k} = \frac{\delta Y_{l,t+n}}{\delta \epsilon_{k,t}} \tag{4}$$

In general, the *(l,k)*-th element of $[\Psi_n]_{l,k}$ can be interpreted as the impact of a one-unit increase in the *k*-th variable's innovation at time t (i.e. $\varepsilon_{k,t} = 1$) on the value of the *l*-th variable at time *t+n* (i.e. $Y_{l,t+n}$), holding all other innovations at all dates constant.

The impulse response function will therefore be a plot of $\frac{\delta Y_{l,t+n}}{\delta \epsilon_{k,t}}$ for $n = 1,\dots,N$. It describes the response of $Y_{l,t+n}$ to a one-time innovation in $Y_{k,t}$ with all other variables dated t, or earlier, held constant.

3.6. Calculating the ECI

The ECI is calculated by taking the weighted average of the indicators and creating a single index as follows:

$$ECI_t = \sum_{k=1}^{9} w_k X_{k,t}, t = 1, 2, 3, ..., T \tag{5}$$

where w_k are the weights for each indicator. These weights are calculated by first estimating a VAR model based on real GDP and each indicator (variable group) listed in Table 1 as shown in equation (2). Then, IRFs are constructed and the weights are then calculated by averaging the twelve-period ahead impulse response of the RGDP variable to shocks from all indicators (including the response of RGDP to a RGDP shock). Therefore, the weights for each indicator are calculated as follows:

$$w_k = \frac{1}{12}\left[\sum_{n=1}^{12} \Psi_n]_{l,k})\right] \qquad (6).$$

[3] Refer to Hamilton (1994) and Greene (2002) for a full description of IRFs in VARs.

4. Empirical results

4.1. VAR estimation results for RGDP equation

Table 2 shows the results obtained from the VAR model described in section 3.4, specifically the results of equation (2). Given that VAR models results are sensitive to lag length, it is important to ensure appropriate lag length via an objective measure. Here we elected to use the AIC which indicated a lag length of 4. The results presented in Table 2 are then used to calculate the IRFs.

Table 2 VAR Estimation Results:

Endogenous variables: RGDP, Macro_Ind, Lend_Ind, Dep_Ind, Credit_Ind, ER_Ind, Ext_Ind, Wealth_Ind, Debt_Ind Deterministic variables: constant Estimation results for equation RGDP			
Regressors	**Coefficients**	**Standard error**	**t-statistics**
Constant	0.0110846	0.0342330	0.324
$RGDP_{t-1}$	0.9533766	0.1372463	6.946***
$RGDP_{t-2}$	-0.3297719	0.1805942	-1.826*
$RGDP_{t-3}$	-0.0172152	0.1771817	-0.097
$RGDP_{t-4}$	0.0705312	0.1272611	0.554
$Macro\_Ind_{t-1}$	0.0249171	0.0633453	0.393
$Macro\_Ind_{t-2}$	0.0416806	0.0700962	0.595
$Macro\_Ind_{t-3}$	-0.0002241	0.0667155	-0.003
$Macro\_Ind_{t-4}$	0.0454718	0.0635500	0.716
$Lend\_Ind_{t-1}$	-0.0366410	0.0582563	-0.629
$Lend\_Ind_{t-2}$	-0.0333573	0.0609974	-0.547
$Lend\_Ind_{t-3}$	-0.1030030	0.0602389	-1.710*
$Lend\_Ind_{t-4}$	-0.0222411	0.0538402	-0.413
Dep_Ind_{t-1}	0.0985294	0.0758424	1.299
Dep_Ind_{t-2}	0.0168802	0.0929955	0.182
Dep_Ind_{t-3}	-0.0851508	0.0872733	-0.976
Dep_Ind_{t-4}	-0.0589893	0.0745533	-0.791
$Credit\_Ind_{t-1}$	0.0827677	0.1046168	0.791
$Credit\_Ind_{t-2}$	-0.0067849	0.1337422	-0.051
$Credit\_Ind_{t-3}$	-0.1884854	0.1339640	-1.407
$Credit\_Ind_{t-4}$	0.1142068	0.0928588	1.230
ER_Ind_{t-1}	0.0604864	0.0420120	1.440
ER_Ind_{t-2}	-0.0901590	0.0629753	-1.432
ER_Ind_{t-3}	0.0334049	0.0628244	0.532
ER_Ind_{t-4}	-0.0364368	0.0486233	-0.749
Ext_Ind_{t-1}	0.0213520	0.0387985	0.550
Ext_Ind_{t-2}	0.0019081	0.0396435	0.048
Ext_Ind_{t-3}	0.0235528	0.0406639	0.579
Ext_Ind_{t-4}	-0.0602003	0.0381687	-1.577
$Wealth\_Ind_{t-1}$	-0.0899304	0.0471020	-1.909*
$Wealth\_Ind_{t-2}$	-0.0665189	0.0384143	-1.732*
$Wealth\_Ind_{t-3}$	-0.0359152	0.0366301	-0.980
$Wealth\_Ind_{t-4}$	0.0746755	0.0355962	2.098**
$Debt\_Ind_{t-1}$	0.1617548	0.1468740	1.101
$Debt\_Ind_{t-2}$	0.2907904	0.2025260	1.436
$Debt\_Ind_{t-3}$	-0.1282845	0.2000195	-0.641
$Debt\_Ind_{t-4}$	-0.3519107	0.1425612	-2.468**
R-Squared: 0.9411 $\bar{R}$-Squared: 0.9045 Residual standard error: 0.3168 F-statistic: 25.74***			

Note: ***, ** and * indicate the rejection of the null hypothesis at the 1%, 5% and 10% levels respectively.

Diagnostic tests on the VAR(4) model were performed and are presented in Appendix 2. As Table C shows, Asymptotic Portmanteau test for autocorrelation reveals that the VAR model has no serial correlation of any order up to 20 lags. Further, the ARCH Heteroscedasticity Test reveals that the residuals of the model may be assumed to be homoskedastic. The Jarque-Bera test for normality, suggests that the residuals have a normal distribution.

4.2. Calculating ECI weights based on IRFs

Following the estimation of the VAR model, IRFs are constructed (refer to appendix 3 for details on the results of the IRFs) so as to construct the weights for the ECI[4]. Table 3 presents the 12-quarter ahead IRF average, calculated weights and the relative importance of each indicator used in the construction of the weighted ECI (which are generated using the methodology discussed in section 3.5). As previously stated, the weights are calculated as the average of the 12-period ahead impulse response of real GDP to innovations in the RGDP, Macro, Lend, Dep, Credit, ER, Ext, Wealth and Debt indicators. Note that some of the indicators have negative weights given that the response of GDP to shocks in certain variables will be negative. In order to generate the relative importance of each indicator, the absolute value of the weights were used.

Table 3 Group weights and relative importance based on 12 quarter IRF averages

Variable groupings	12-qr IRF average	Weights	Relative importance (%) of each variable
RGDP	0.4099	0.645	28.55
Macro	0.0909	0.143	6.33
Lend	-0.1205	-0.189	8.39
Dep	-0.0892	-0.140	6.21
Credit	-0.1062	-0.167	7.39
ER	-0.0188	-0.030	1.31
Ext	-0.0654	-0.103	4.55
Wealth	0.0076	0.012	0.53
Debt	0.5274	0.829	36.73

As table 3 shows, the debt indicator has the highest weight, followed by RGDP and the lending rates indicator. Together, these three indicators have a relative

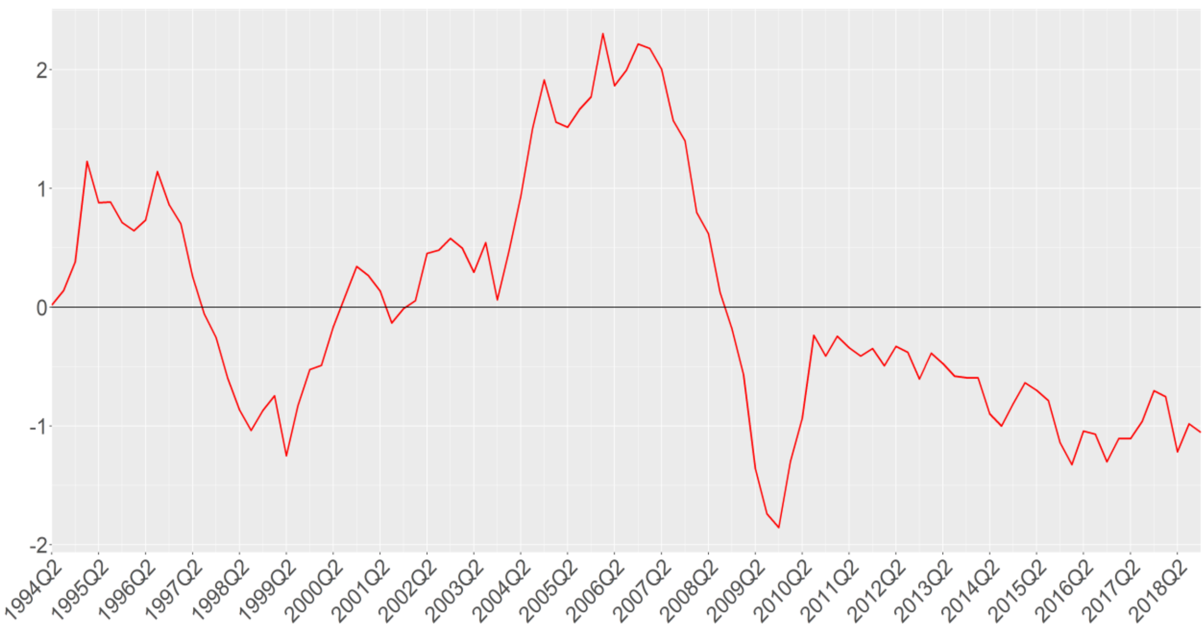
[4] To ensure that the impulse responses are invariant to the ordering of the variables in the underlying VAR, the paper uses the "generalized impulse response analysis" developed by Pesaran and Shin (1998).

importance of 73.67%. Therefore, movements in these indicators will have the largest impact on the ECI. The least important indicator is the wealth indicator, with a relative importance of 0.53%.

4.3. Economic conditions index results

Given the indicators and the weights calculated, the ECI can be constructed using equation (5). Periods where the ECI is above 0 indicate favourable economic conditions in South Africa, based on the indicators included in the construction of the index. Inversely, for periods where the indicator is less than 0, economic conditions in South Africa are relatively worse. The results are shown in Figure 2.

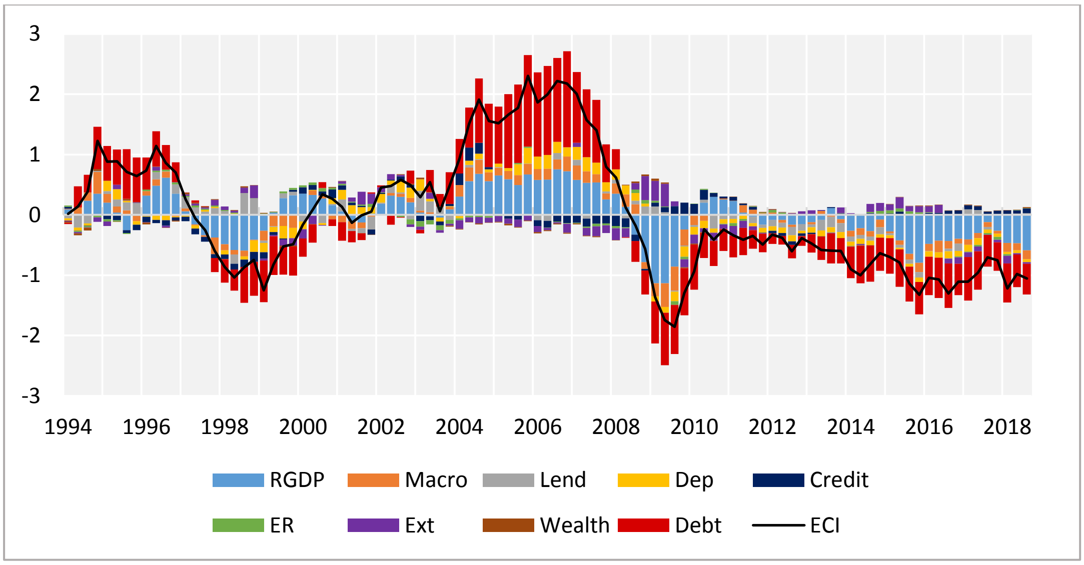
Figure 2 Economic Conditions Index for South Africa



As the figure shows, the ECI picks up the effects of the Asian financial crisis and subsequent ruble crisis from 1997. These crises led to the deterioration in South Africa's income growth and employment, in an environment of high interest rates, thus affecting economic growth, wealth levels and the exchange rate. The ECI also decreased to below zero in the second half of 2001, affected by both the bursting of the dotcom bubble and subsequent decline in economic activity in advanced economies and the effects of the 2001 rand crisis. A severe deterioration is also observed during the 2008 Global Financial Crisis (GFC). This decrease was also driven by a drastic fall in income and wealth, with almost all indicators deteriorating. More recently, South Africa's ECI has failed to recover from the steep drop during the 2008 GFC, remaining below zero in the period thereafter. The ECI has also been on a gradual downward trend since the end of 2010. This steady decline has been driven by similar downward trends in the debt indicator, RGDP, the macroeconomic indicator and the credit indicator.

Figure 3 shows a plot of all the indicators (multiplied by their individual weights) in order to depict a decomposition of the ECI and show how each indicator contributes to the ECI. The figure shows that, as expected, the debt indicator has contributed the most to the movement in the ECI throughout the sample period. But its effect is especially prominent during the build up to the 2008 GFC. Real GDP, which is the biggest contributor to the deterioration of the ECI during the 2008 GFC, has been one of the main drivers, along with the debt indicator, of the gradual decrease in the ECI since 2010. More recently, the credit and lending rates indicators have been positive contributors to the ECI, but with a fairly limited impact.

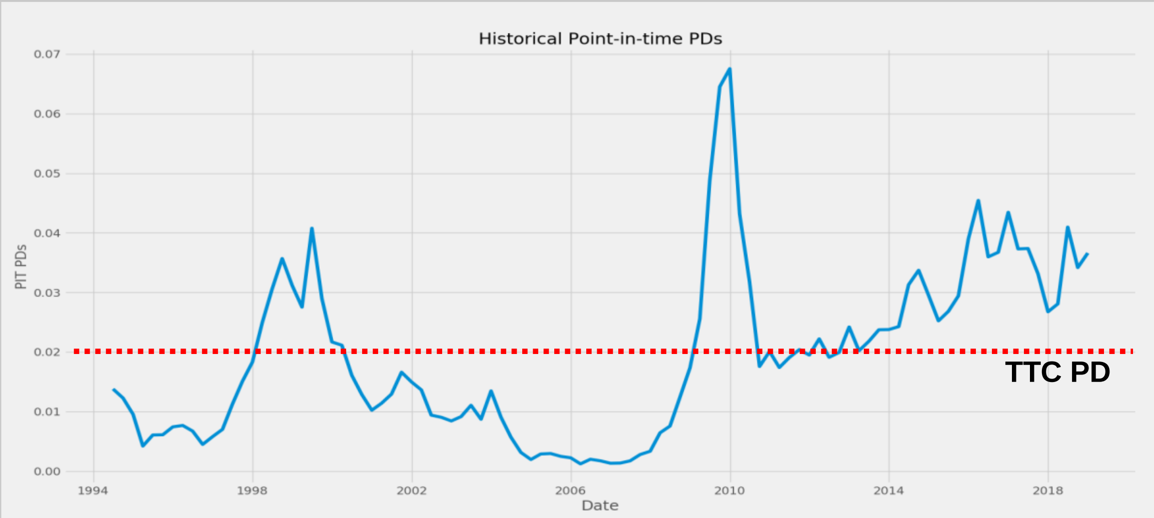
Figure 3 Decomposition of the ECI



4.4. PIT PD Example

Given the economic conditions index above we are now able to transform the TTC PDs into PIT PDs. For an illustrative portfolio that has TTC PD of 2%, asset correlation of 0.12 and given the ECI derived above, the historical PIT PDs is depicted by the graph below:

Figure 4: Historical PIT PDs for a fictitious portfolio with TTC PD of 2%



5. Conclusion

Banks are vital for financial intermediation and safeguarding their resilience in the worst macroeconomic scenarios is critical. Stress testing plays a critical role in ensuring that both banks and regulators accurately assess latent vulnerabilities in the banking sector. Intending to contribute towards that outcome, this paper highlights the need for models that are appropriately responsive and adequately measure the impacts of macroeconomic shocks to the performance of the banks.

This paper seeks to expand on the discussion of the appropriate methodology to measure the risk parameters used for credit stress testing. While the literature shows unanimity in the use of the Vasicek single risk factor model for the transformation of TTC PDs into PIT PDs, there is lack of consensus with regards to the appropriate way to model a measure of the systematic risk factor. The current contribution is the use of the Vector autoregressive model to calculate the economic cycle index. Using South African data, we observed that the model gives a realistic estimate of the cycle index which appears to capture major historical events.

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7. Appendices

Appendix 1 Unit root test results

Stationarity tests are conducted using the Augmented Dickey-Fuller unit root test (ADF) suggested by Said and Dickey (1984). The ADF test is carried out in the same way as the DF test but is applied to a model that includes the lagged values of the dependent variable so as to control for serial correlation in the error term. When the null hypothesis is rejected then no unit root is present, and one can conclude that the variable is stationary.

Table A and B present the results for ADF unit root tests for the variables and the indicators respectively. It is evident from Table A that the ADF tests conducted, both with and without a drift, reject the null hypothesis of the presence of unit roots in all the variables at either their levels or first difference with or without drift with a level of significance of at least 5%. In terms of the indicators, Table B shows that the null hypothesis of non-stationarity for all the indicators are rejected at their levels without drift. When the test is done with a drift, however, we fail to reject the null hypothesis for the Dep and Debt indicators.

A rule of thumb for this paper is to exclude any variable or indicator that is non-stationary after first differencing. Therefore, based on the fact that the null hypothesis is rejected for all the variables either with or without trend or for both cases, at their levels or after first differencing, none of the variables or the indicators are excluded from either the calculation of the indicators using PCA or the VAR model.

Table A Augmented Dickey-Fuller (ADF) unit root test on variables

Variable	With drift		Without drift	
	Level	1st Diff	Level	1st Diff
Real GDP	-3.6829**		-3.7034**	
CPI	-3.5542**		-3.5665**	
Non-agricultural employment	-3.4400**		-3.4583**	
Unemployment	-2.8590	-8.8519**	-2.7113**	
Consumption expenditure	-4.0009**		-4.0193**	
Investment	-3.5654**		-3.5739**	
M3	-1.8718	-10.5966**	-1.8777	-10.6281**
Repo	-1.5199	-6.3700**	-1.5178	-6.3566**
Non-mortgage lending rate	-1.6535	-8.8033**	-1.6406	-8.8089**
Mortgage lending rate	-1.8332	-5.9493**	-1.8293	-5.9431**

Long term gov. bond yields	-2.3317	-8.5513**	-2.2183**	
Short term credit spread	-3.9000**		-3.9174**	
Long term credit spread	-2.4434	-8.0185**	-2.4508**	
Short term deposit rate	-2.0728	-6.5477**	-2.0680**	
Long term deposit rate	-2.2648	-6.4319**	-2.2501**	
Short term deposits	-3.8434**		-3.8600**	
Long term deposits	-3.6457**		-3.6663**	
Broad credit	-2.8730	-4.1976**	-2.8989**	
Private sector credit	-2.1925	-9.1178**	-2.1930**	
Non-mortgage credit	-3.3265**		-3.3437**	
Mortgage credit	-2.5033	-2.7118	-2.5206**	
NEER	-3.1110**		-3.1277**	
Dollar/Rand	-4.0750**		-4.0973**	
Dollar/Euro	-3.2974**		-3.3090**	
Oil prices	-4.9048**		-4.9301**	
Commodity prices	-5.8327**		-5.8641**	
Foreign wholesale prices	-5.5766**		-5.6068**	
Global growth	-5.4947**		-5.5251**	
Current account to GDP	-2.0341	-10.4043**	-2.0372**	
All share index	-4.2056**		-4.2140**	
House Price index	-1.4419	-6.4323**	-1.4527	-6.4684**
Savings to Disposable income	-1.4120	-17.3774**	-1.3794	-17.4159**
Disposable income	-2.5314	-9.0868**	-2.5532**	
Household wealth	-2.9701**		-2.9871**	
Household assets	-2.4591	-7.3230**	-2.4713**	
Household debt to Disposable income	-1.7204	-2.8553	-1.7206	-2.8696**
Household debt to GDP	-1.4543	-3.4955**	-1.4502	-3.5173**
Household debt	-3.3840**	-3.3840	-3.3966**	
Gov. debt to GDP	-1.1685	-1.9900	-1.2004	-1.9750**
Household financial Liabilities	-3.4071**	-3.4071	-3.4197**	

Notes: Figures denote t-statistics. ** indicates the rejection of the null hypothesis at the 5% level. Lag lengths were chosen using Bayesian information criterion (BIC) to be a maximum of 3 for all variables.

Table B Augmented Dickey-Fuller (ADF) unit root test on indicators

Variable	With drift	Without drift
	Level	Level
Real GDP	-3.633**	-3.6524**
Macro	-2.9093**	-2.8837**
Lend	-6.6911**	-6.7252**

Dep	-2.7036	-2.7137**
Credit	-3.2134**	-3.2552**
ER	-4.1361**	-4.1567**
Ext	-4.2629**	-4.2861**
Wealth	-3.7511**	-3.7619**
Debt	-2.5391	-2.5460**

Notes: Figures denote t-statistics. ** indicates the rejection of the null hypothesis at the 5% level. Lag lengths were chosen using Bayesian information criterion (BIC) to be a maximum of 1,0,0,0,5,1,0,4,3 for Real GDP, Macro, Lend, Dep, Credit, ER, Ext, Wealth, Debt respectively.

Appendix 2 Diagnostic tests

Table C Diagnostics tests

Tests for normality, autocorrelation, heteroscedasticity			
Test	**H_0**	**χ^2**	**Prob**
Asymptotic Portmanteau Residual Test for Autocorrelation	**No residual autocorrelations up to lag 20**	1294.9	0.5037
ARCH Heteroscedasticity Test	**Residuals are homoscedastic**	3305.3166	0.2077
JB Residual Normality Test	**residuals are multivariate normal**	17.515	0.488

Notes: ** indicates the rejection of the null hypothesis at the 5% level. If prob>0.05, fail to reject the null hypothesis (H_0).

Appendix 3 Impulse Response functions